Testing, Optimization, and Games

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The Software Reliability Problem

Systems are becoming larger, more complex,distributed,…

$\Rightarrow$ harder to create, get them right, test them …

- Large part of the cost of software development goes to testing

Problem: Improve cost, time, reliability

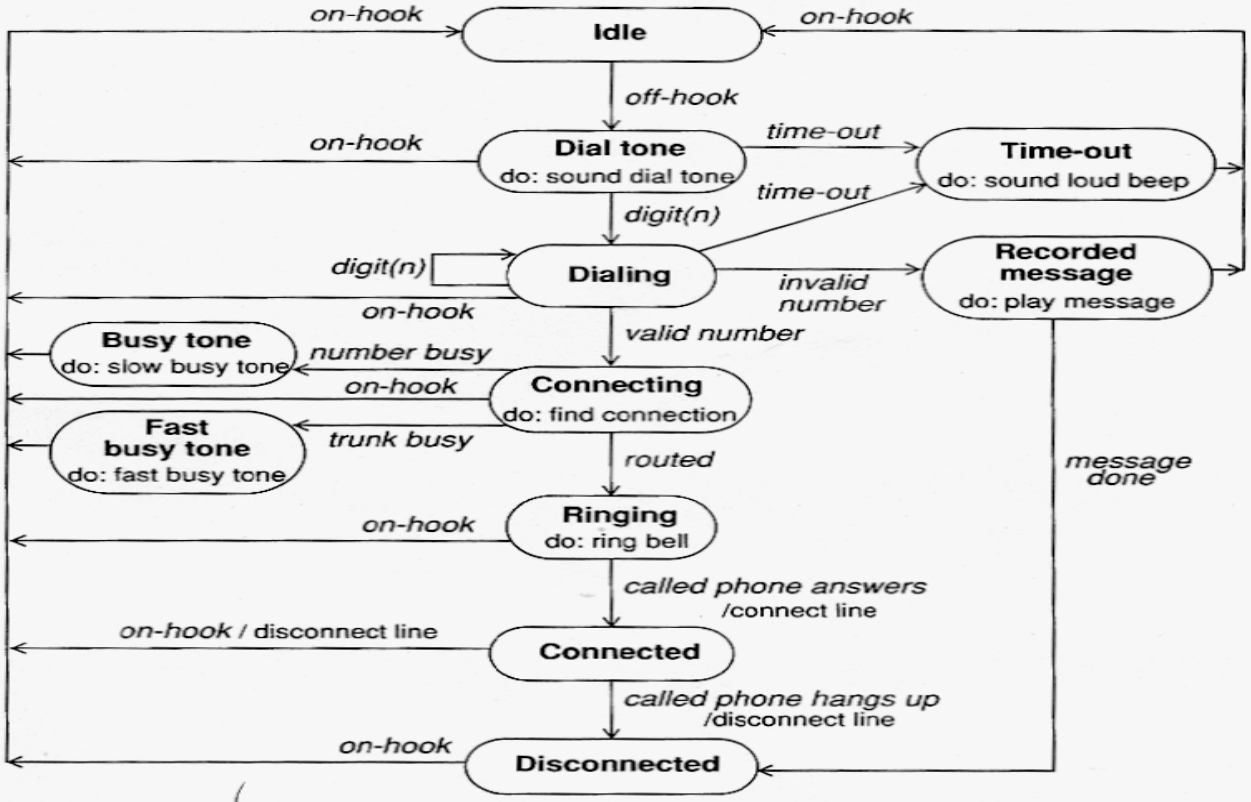
Focus: Behavior/Control of Systems

Reactive/Event-driven Systems

- Switching Software
- Communication Protocols
- Controllers
- ….

Model: State Machines of various types

Finite State Machine for Phone

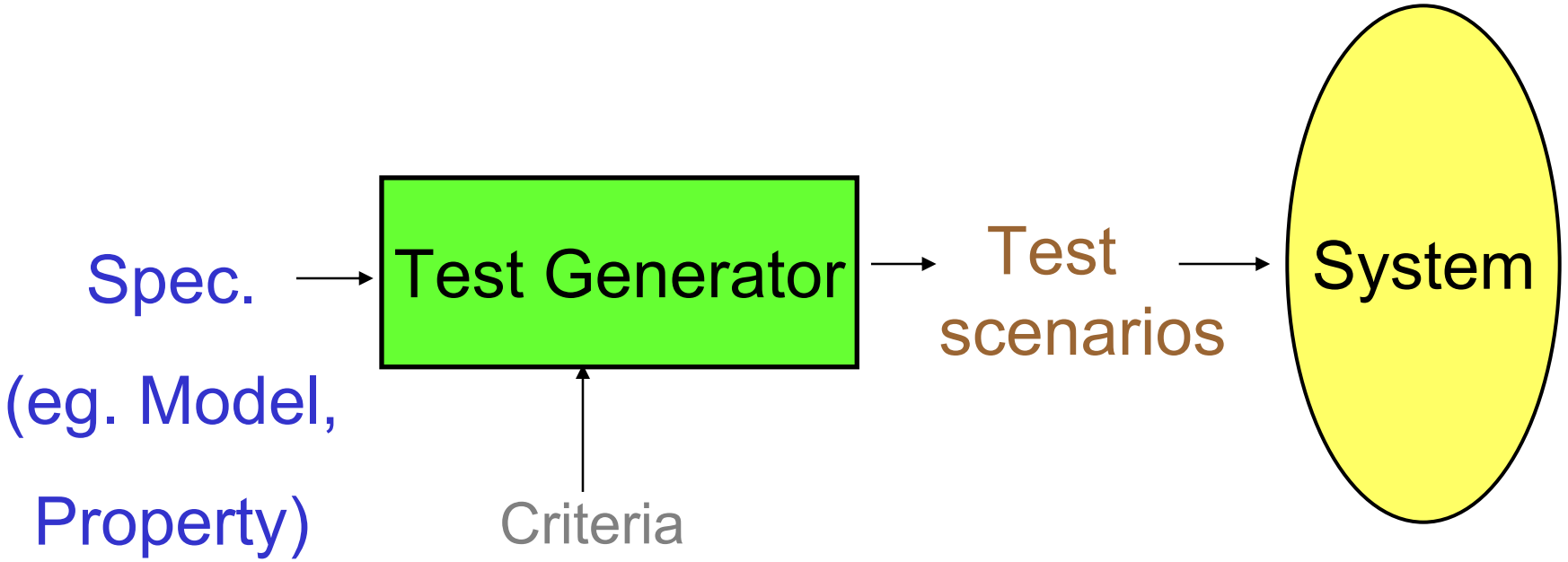


States: Idle, Dial tone, ….

Inputs: off-hook, on-hook, digit, …

Outputs: sound dial tone, loud beep, play message,….

Testing



Does the System satisfy the specification?

(conform to the model ? satisfy the property?)

Different Views of Testing

- Testing as an Optimization problem

  Optimize the use of testing resources to achieve maximum fault coverage

- Testing as a Game

  Tester vs. System

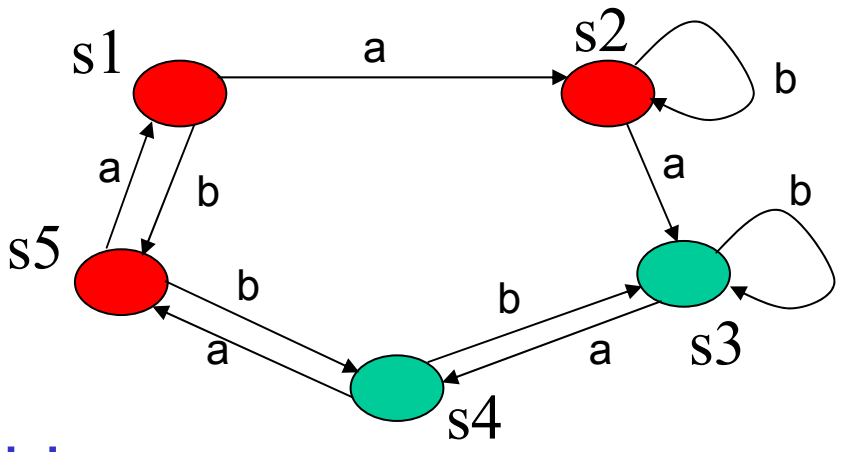
  Who wins? Best strategy?

- Testing as a learning problem

Outline

- Testing framework, issues
- Conformance Testing
  - Deterministic FSM's
  - Nondeterministic FSM's
- Testing Properties
- Optimum Coverage problems
  - FSM's, graph models
  - Extended FSM's
  - Hierarchical FSM's

Finite State Machine



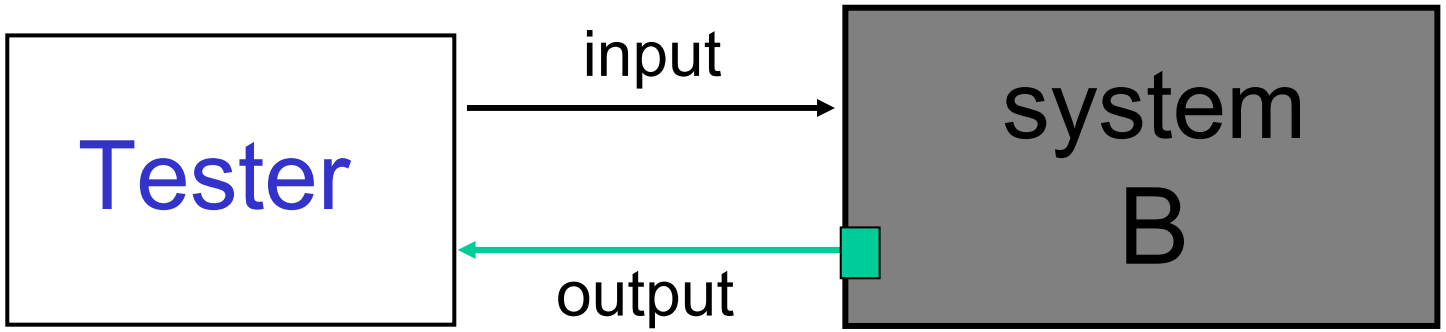
Moore machine

- States: s1, …., s5
- Inputs: a, b
- Outputs: red, green - function of the state
- Transitions: for every state and input

Deterministic FSM: one transition for every state and input

Mealy machine: variant where outputs are produced on transitions instead of states; theory is similar

Test



Problem: Given some a priori information about B, compute a desired function of B

Preset Test: input sequence selected ahead of time

Adaptive Test: inputs selected online adaptively,
i.e. can depend on previous outputs

Testing as a Game

- Game:

  1. A priori information ("testing hypothesis"): Set U of possible B's

  2. Desired information: function f of B

- Players:

  - Tester: selects inputs, gives verdict at end

  - System: Selects B in U, and moves of B in each step (if B not deterministic)

- Tester wins if verdict=f(B)

- Game with incomplete information

Questions

- Can the Tester always win?

  i.e. ∃ strategy (test) that arrives at correct result?

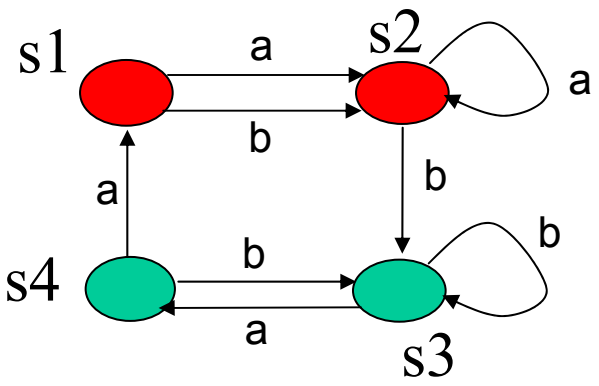
- How fast can we determine if the Tester has a winning strategy?

- What is the *testing complexity* = length of the test (winning strategy)

- and the *computational complexity* = time to compute a winning strategy?

Example: Adaptive Distinguishing "Sequence"

Given:

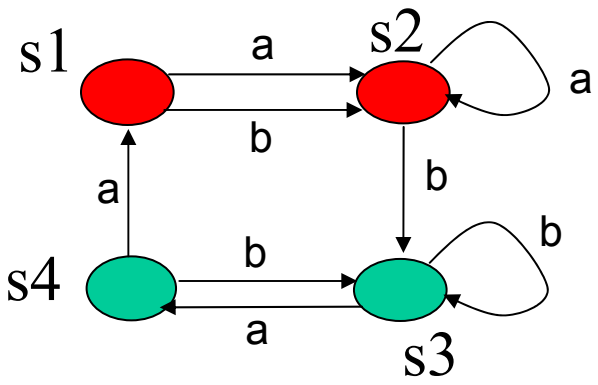


State diagram of B =

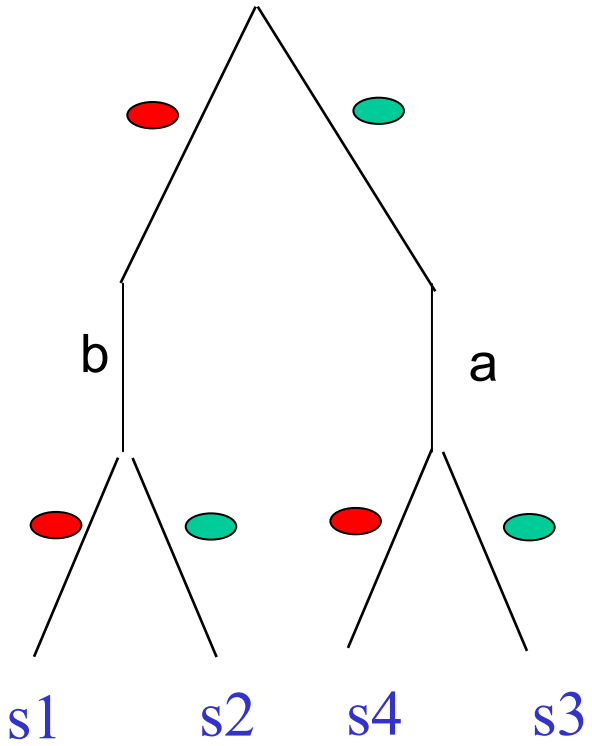
a deterministic FSM

Goal: Determine the initial state of B

Example: Adaptive Distinguishing “Sequence”



FSM

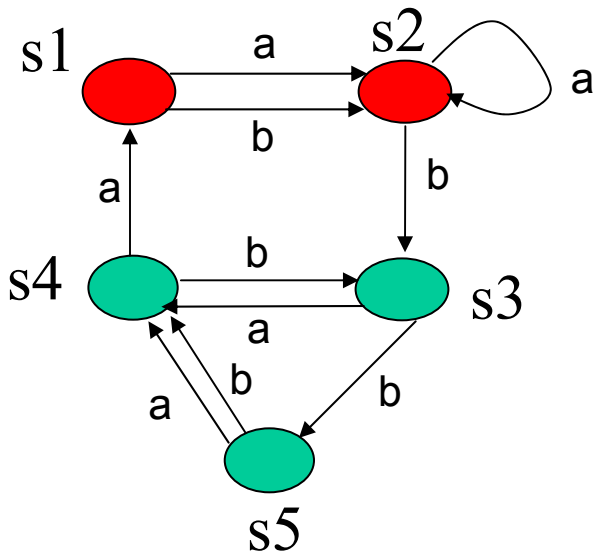


adaptive distinguishing “sequence”

= winning testing strategy

Questions

- Can the Tester always win?
  - No (not even if FSM is reduced, i.e. has no equivalent states)



Questions

- Can the Tester always win?
  - No (not even if FSM is reduced, i.e. has no equivalent states)
- How fast can we determine if the Tester has a winning strategy?
  - O(dnlogn), n=#states, d=#inputs
  - For Preset test: PSPACE-complete

Questions

- Can the Tester always win?
  - No (not even if FSM is reduced, i.e. has no equivalent states)
- How fast can we determine if the Tester has a winning strategy?
  - O(dnlogn), n=#states, d=#inputs
- What is the *testing complexity* = length of the test (winning strategy)
  - $O(n^2)$
- and the *computational complexity* = time to compute a winning strategy?
  - $O(dn^2)$
- Preset: Exponential

[Lee-Yannakakis]

Unknown state diagram of black box B

- Machine Identification Problem:
- Given:
- B is a reduced (minimized) deterministic FSM

  (tests cannot tell the difference between equivalent machines)

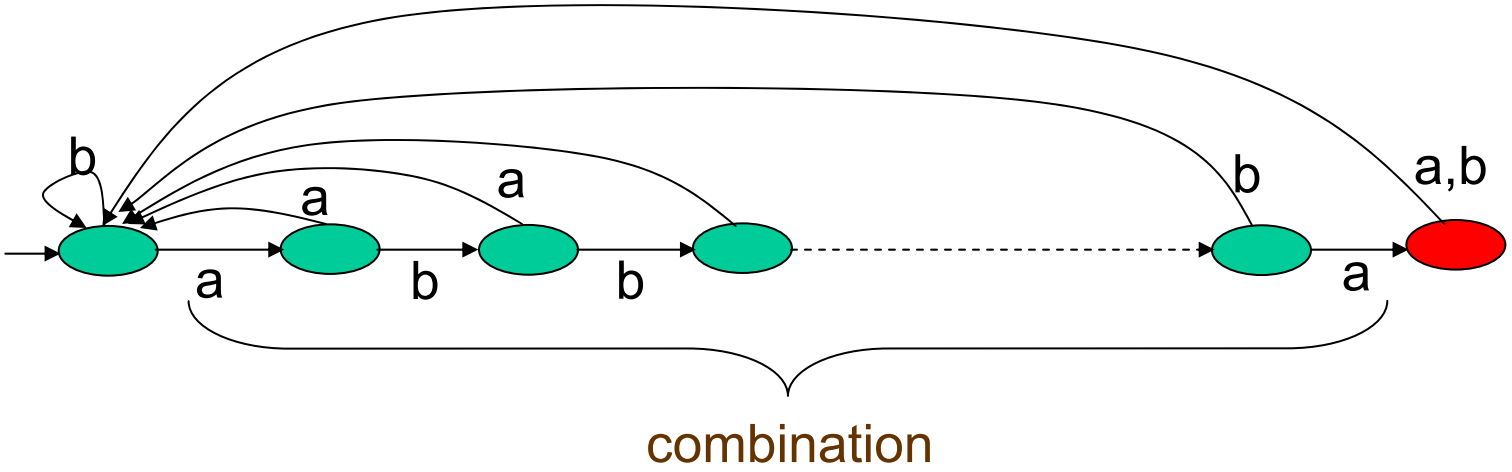
  - and strongly connected

  (i.e. any state can reach any other state)
- bound on # states of B

Goal: Identify machine B

Machine Identification is hard

- Suppose that we know B has *n* states and looks like this combination lock machine



Must try all possible combinations: d^{n-1}

d = # inputs, n = # states

[Moore]

Conformance Testing

- Given: specification FSM A

- Goal: check that B conforms to (behaves like) A

  (i.e. B≡A for deterministic FSMs)

- Long History since 50's [Moore, Hennie,…]

Conformance Testing - Deterministic FSM

Assumptions

- Specification machine A is reduced (minimized)

  (tests cannot tell the difference between equivalent states)

  and strongly connected

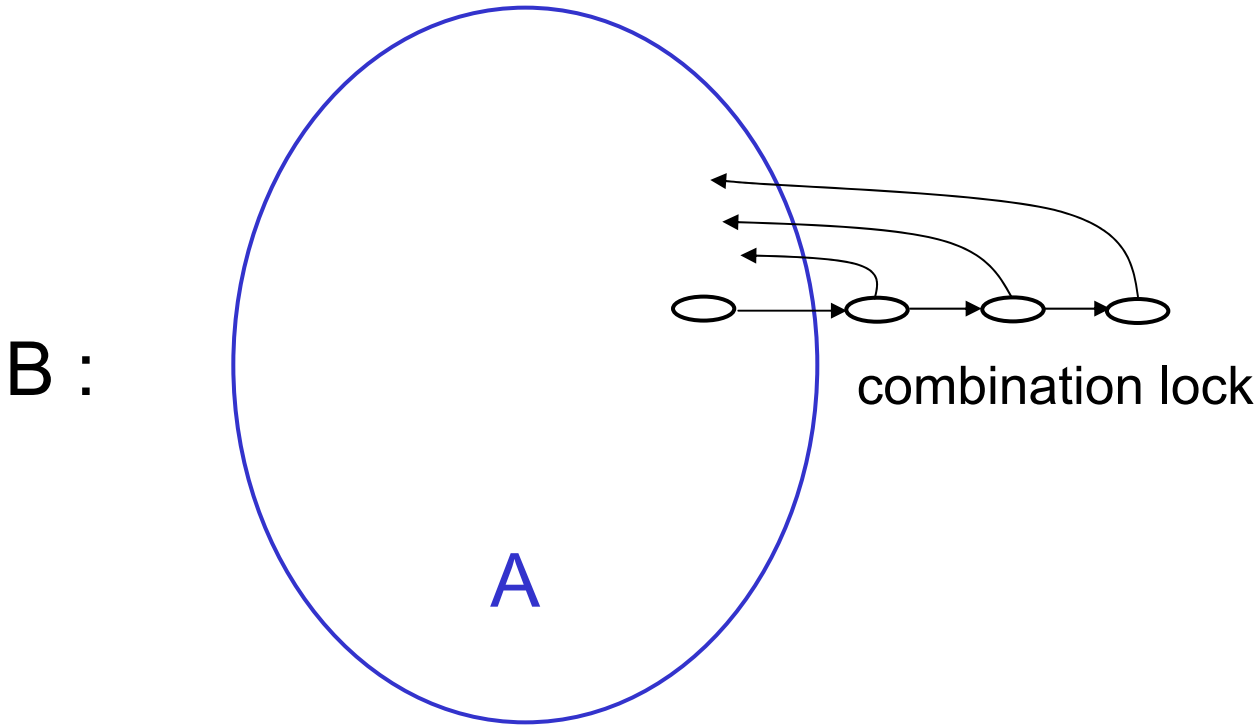
  (i.e. any state can reach any other state)

- Bound on #states of B
- Checking sequence: If implementation machine B has no more states than A: detect arbitrary combinations of *output*, and *next-state* faults

  - effect of extra states orthogonal

Effect of extra states

Extra factor of d^k, where k =#extra states, d=# inputs



Questions

- Can the Tester always win?
  1. Can test that B has the same state diagram as A
  2. But in general may not be able to verify the initial state (if no reset) even if we know state diagram of B

- Can perform a test such that if B passes it, then can conclude that B≡A and B is at an equivalent state *at the end* of the test

Easy cases

- Spec FSM A is fully observable:

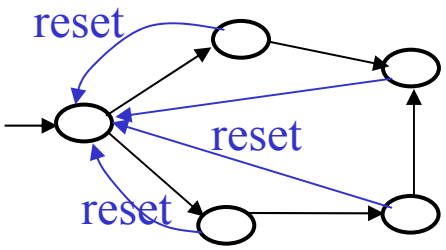
  every state has a distinct output $\Rightarrow$ suffices to traverse all the transitions

- Spec FSM A has a distinguishing sequence:

  $\Rightarrow$ checking sequence of length $O(dn^3)$

[Hennie,LY]

Machines with Reliable Reset



- There is a special input symbol "*reset*" which takes every state back to the initial state
- *Reliable*: works properly in the implementation FSM B

- Then checking sequence of length $O(dn^3)$
- Matching lower bound

[Vasilevski- Chow]

General machines

- Randomized polynomial time algorithm which, given a specification machine A constructs with high probability a checking sequence for A of length $O(dn^4 \log n)$ [LY]
- For "almost all" specs A, length O($d \cdot n \cdot$polylog$n$)
- Deterministic algorithm?

Sketch of (simplified) Test

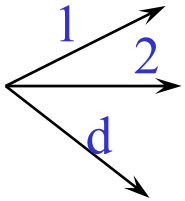
- Pick a set W of "separating" input sequences such that every pair of states of the spec FSM A is distinguished by one of these sequences
  - There is always such a set of at most n sequences of length at most n

Repeat the following "enough" times

- Choose at random a transition (state s, input *a*)
- Apply an input sequence that takes A from the current state to state s
- Decide at random whether to check the state of B or check the transition
  - In the first case, apply a random separating sequence from W
  - In the second case, apply input *a* followed by a random separating sequence from W

A universal traversal problem

Directed graphs with n nodes, outdegree d



- Blocking sequence over {1,...,d}:
  For every graph and starting node,
  path traverses all edges out of at least one node.

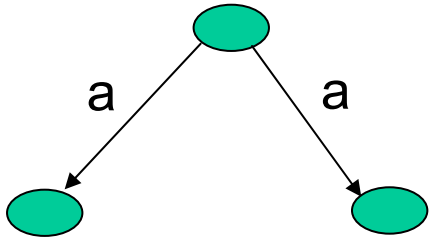
- Random sequences of polynomial length blocking

- Deterministic polynomial construction?

Then deterministic construction of checking sequence for all spec FSM's

Nondeterministic FSM

Many possible transitions for same input and state

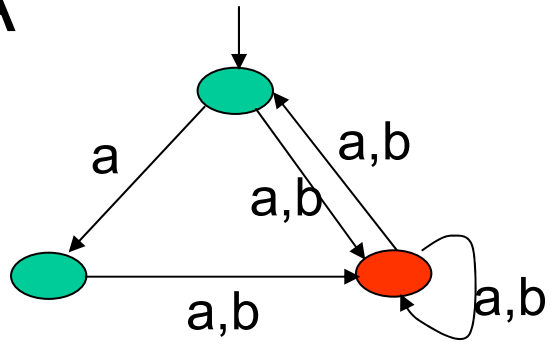


- Nondeterminism in spec A: multiple acceptable choices
- Nondeterminism in system B: some transitions are not under tester's control
  - abstraction, other entities, concurrency, ..

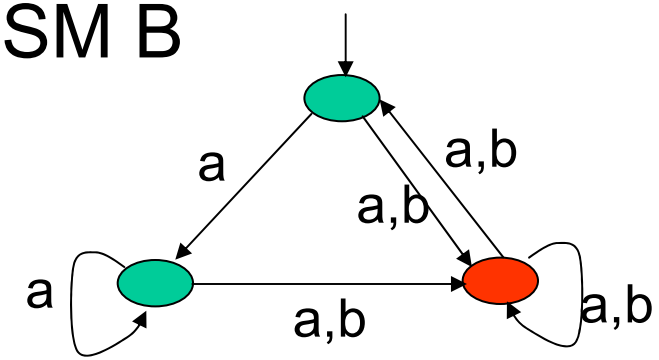
FSM B conforms to FSM spec A if every response to any input sequence could have been produced by A

Example

Spec A



FSM B

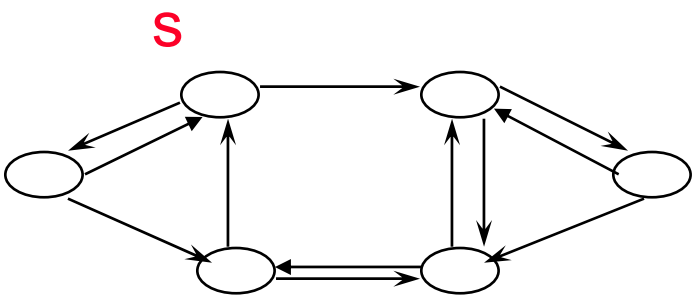


- B does not conform to A:
  On input aa , B *may* output ● ● ●, but not A
- B may also output ● ● ● or ● ● ● or ● ● ● which are consistent with A

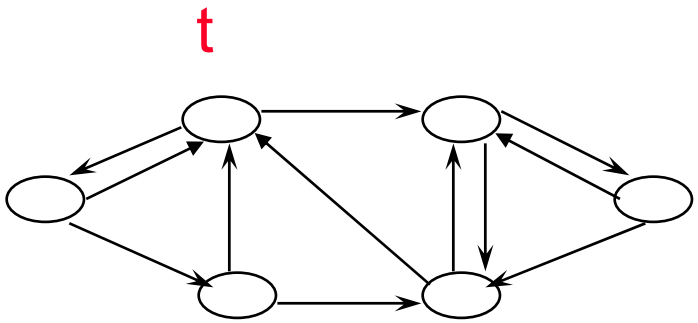
Distinguishing Between Machines

Spec A

(correct FSM)



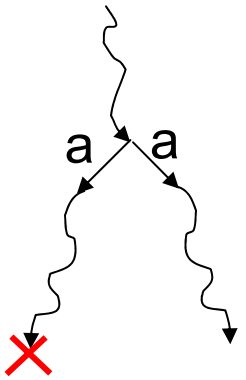
Possible faulty FSM B



Two-player game

- Tester chooses inputs
- System player chooses what's in the black box and *how to resolve the nondeterminism*

- Should we view the system player as trying to
  - Help the tester?
  - Oppose the tester?
  - Indifferent (random)?



Opposing System Player

- Tester has winning strategy $\Leftrightarrow$ can find a fault (if present) no matter how hard the system tries to hide it

$\Leftrightarrow$ Games with incomplete information against a malicious adversary

- Game graph of positions, controlled by the two players
- Player 1 gets only partial information about current position
- Goal of Player 1: reach a winning position

Who wins?

- preset test: PSPACE-complete
- adaptive test: EXPTIME-complete
- Polynomial time for NFSM that are input-output deterministic (observable)

[Reif; Alur, Courcoubetis, Y]

Indifferent System player: Random moves

If the system has *reliable reset,* then easy: can test with probability $\rightarrow 1$

B does not conform to A $\Rightarrow$ for some input sequence α it can produce (for some nondeterministic path) an output sequence that can't be produced by A

Test: Apply repeatedly reset α , reset α, ….

Indifferent System player: Random moves

In general, Game with incomplete information against "Nature" (a Random adversary)

- Partially observable Markov Decision Process
  - maximize probability of reaching goal
  - can we reach goal a.s.?

Can the Tester win with probability 1 (in the limit)?

- Complexities similar to adversarial game – algorithms different

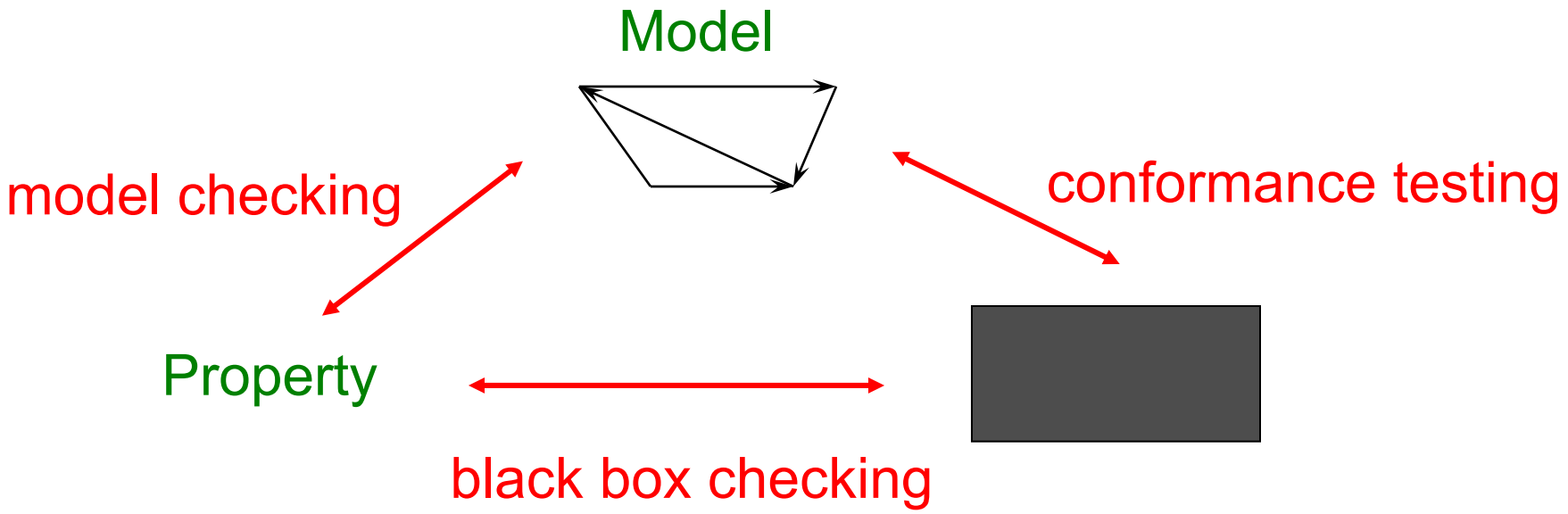
[ACY]

Testing Properties

Testing Properties

- Given a required property of executions
  - e.g., if off-hook then dial-tone; no deadlock …
  - between any two green states always a red state
- and a black box B (the system)

Test that B satisfies the property



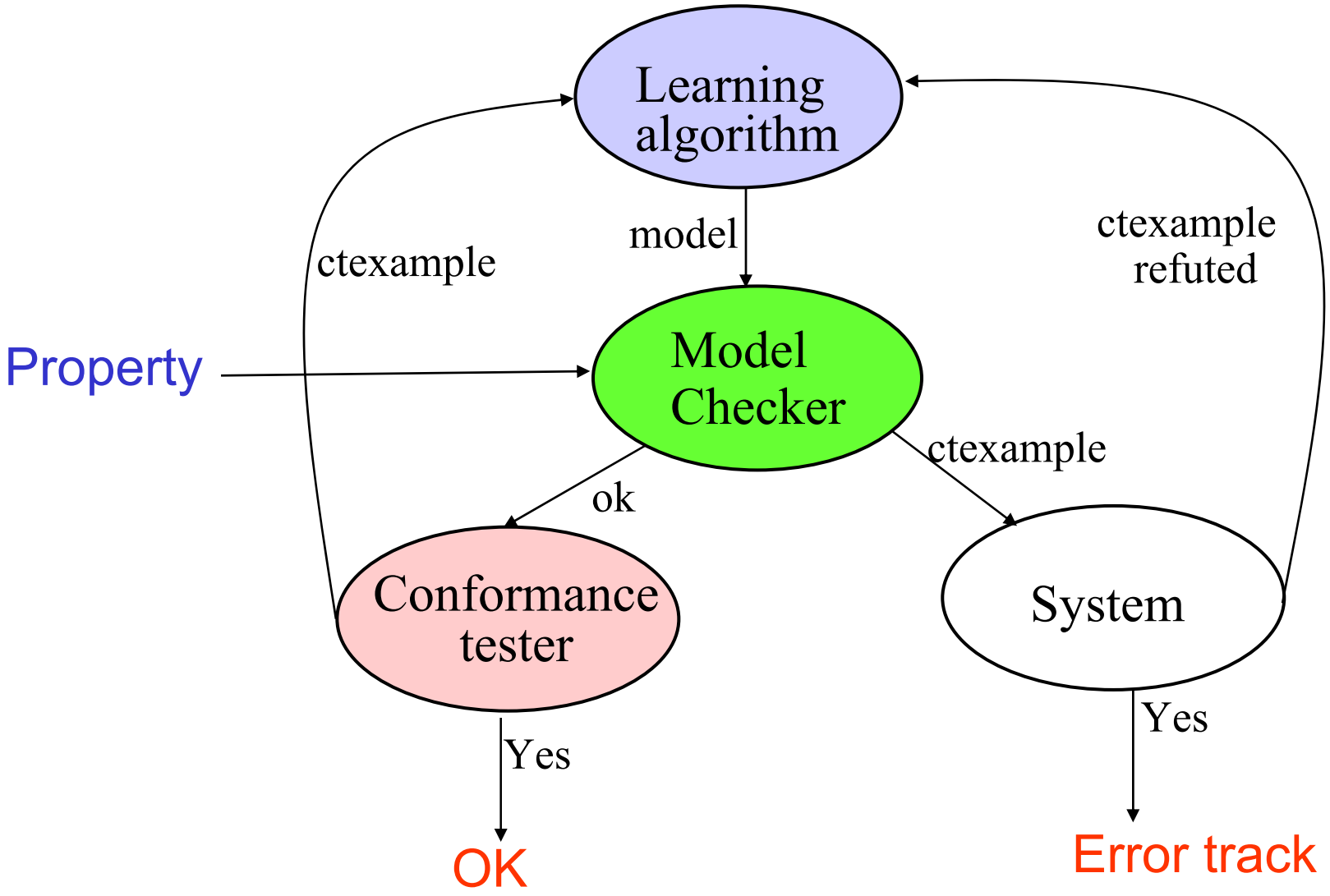
[Peled, Vardi, Yannakakis]

Learning FSM with a teacher

- Algorithm to identify a deterministic FSM using
  - "membership queries" (tests) on the black box
  - "equivalence queries" to the teacher

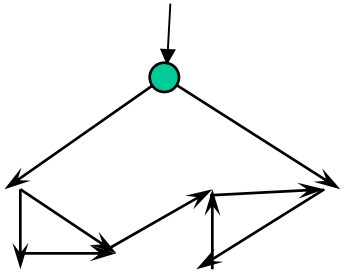
- FSM with reset: polynomial algorithm [Angluin]
- General FSM: randomized polynomial algorithm [Rivest –Shapire]

Black Box Checking



Optimization

Optimal Coverage Problems



- Find a minimum number of short test sequences (paths) starting at initial state that cover all transitions, states .....

- Applies to FSM models and other graph models
- Use Case (MSC) Graphs: scenario based models

  uBET - Lucent Behavior Engineering Toolset

Graph Coverage

- Transition Coverage

  Can minimize in PTIME

  (1) the number of paths,

  (2) their total length, subject to (1)

  (or any linear combination of 1 and 2)

  - Network flows, Chinese Postman Problem

- State Coverage

  Can minimize the number of paths

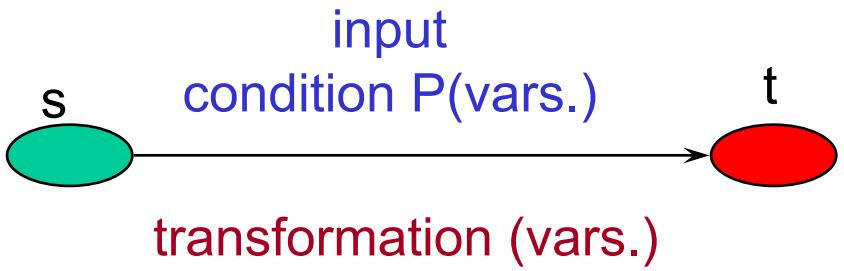
  but not the length

  - Asymmetric Traveling Salesman Problem

Extended Finite State Machine

FSM + variables

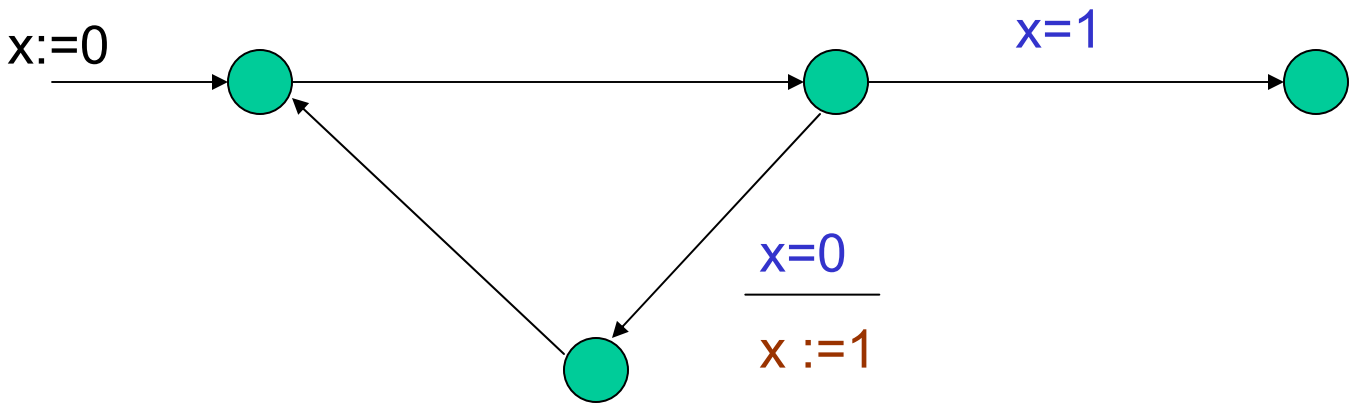
- States
- Variables
  (Boolean, arithmetic, …)
- Transitions



- Initial state, variable assignment

Covering Tests for EFSM

- Find minimum number of valid paths that cover all the transitions of the EFSM



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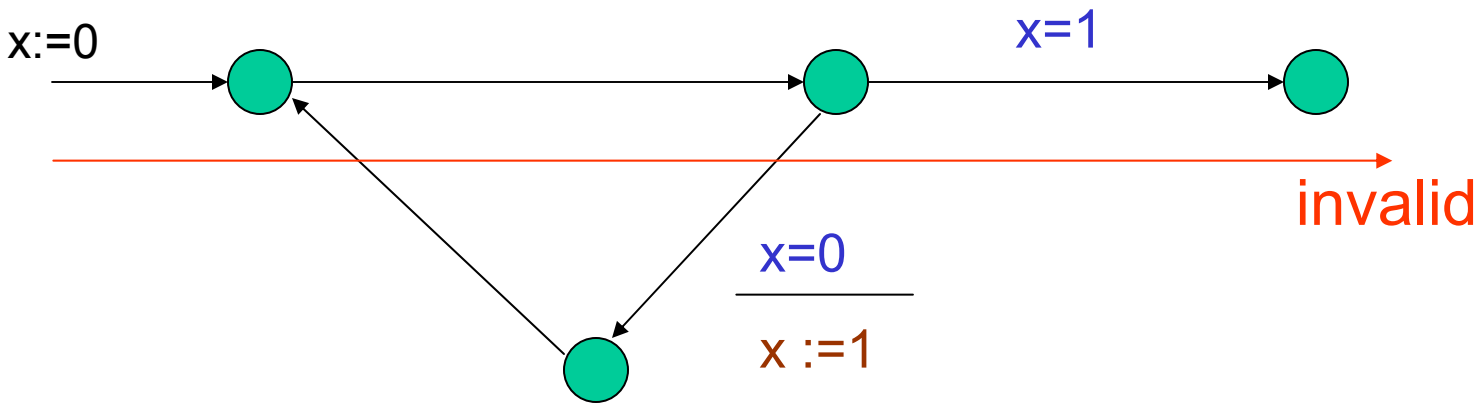

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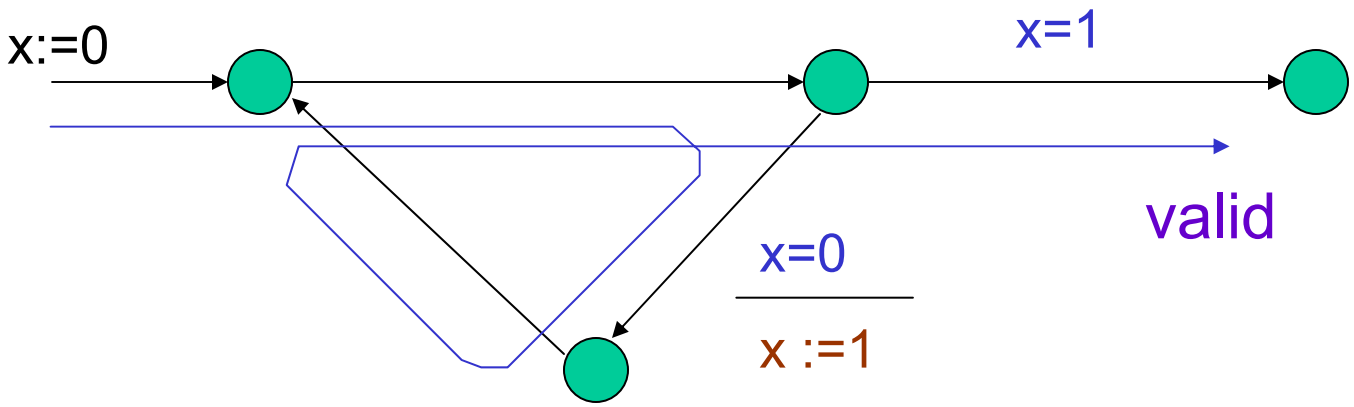
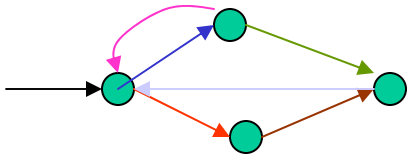


EFSM → Colored Graph

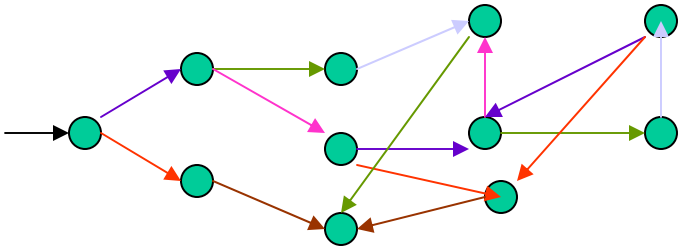
- One color per transition of EFSM

EFSM



Expanded FSM

(no variables)



Find minimum number of paths covering all the colors

Optimization Problems

Given a graph with colored edges

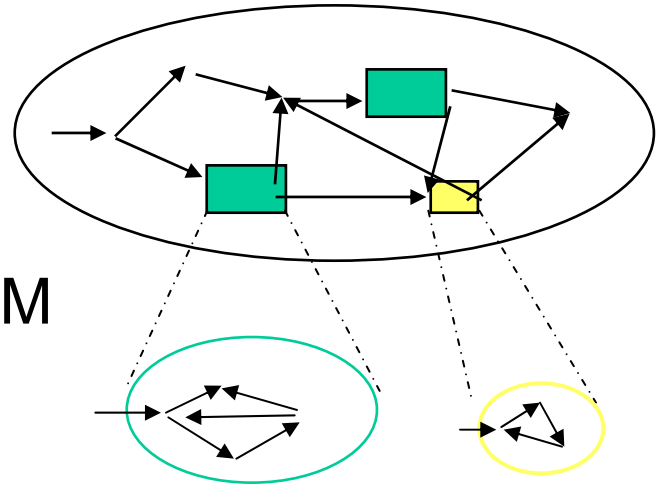
- Find minimum set of paths covering all colors
  - Hard (harder than Set Cover)
- Find a path covering maximum number of colors
  - Still hard
- Find a path covering at least k colors if ∃ (k fixed)
  - Solvable efficiently

Pythia

- Toolset for automated test generation for FSM's and EFSM's (Lee & Yannakakis)
- Incorporates optimization algorithms
- Applications to systems:
  - PHS, 5ESS INAP, Diamond, H.248

Hierarchical FSM

Nodes are ordinary states or superstates mapped to lower level FSMs



Compact representation of large flat FSM

- Useful way to structure large FSM

• Find minimum number of tests to cover all transitions of all the modules

- Could expand to flat FSM and reduce to colored graph covering problem

• Much better: Can avoid flattening and can get constant approximation ratio = nesting depth [Mosk-Aoyama, Yann.]

Conclusions

- Long line of research
- Theoretical and practical interest
- Rich variety of problems
- Connections with different areas (optimization, verification, learning, games, combinatorics,…)